Alberta Buck Notes - Single vs. Serialized B1 Sub-Notes

Perry Kundert

2026-04-23

The Phase 7-bis batch-mint pivot (see `alberta-buck-notes-rollup-mint.org`) pushes per-leaf Merkle insertion into the SNARK and recovers ~N-fold mint amortization at the *batch* level. This memo describes a second, orthogonal amortization that operates *within a single mint leaf*: a B1 bearer-note family whose parent commitment in `noteRoot` anchors a cryptographically committed sub-tree of N sub-notes, each independently spendable against a shared per-parent nullifier bitmap.

The central construction: *1 Note = 1 unique bit in a shared nullifier bitmap*, with N bitmap bits anchored by a single `cm_parent` leaf. Recipient payloads differ only in a 256-bit hidden serial `s_i` and its sub-tree path; the serial position `i` (the bit index) is hidden from outside observers and from sibling recipients but recoverable inside the spend circuit via a witness-and-verify discipline, never by inversion.

Status-quo single-note mints remain the only appropriate construction for identified (A1/A2) flows; serialization is B1-only because each spend reveals the family parent `cm_parent` and thereby links co-spent siblings. For bearer notes that linkage carries no privacy cost: bearer-to-bearer transfer is already a public act.

This memo lays out, side-by-side, the construction, data flow, per-field security analysis, storage layout, cost model, and attack surface of both encodings. Where the mint-batch pivot amortized the cost of publishing N commitments into `noteRoot`, serialization amortizes the cost of producing N bearer notes from *one* commitment. Together the two drop marginal per-note mint cost by 2-3 orders of magnitude vs. the shipped per-leaf design, at the cost of the linkability disclosure described in *Privacy Model*.

Contents

1 Scope and Summary **4**
- 1.1 Applicability . . . 4
- 1.2 Headline differences . . . 4

2 Construction **4**
- 2.1 Single Note (status quo) . . . 4
- 2.2 Serialized B1 sub-notes . . . 4
  - 2.2.1 How PRF-of-index becomes witness-and-verify . . . 5
  - 2.2.2 Mint-circuit reuse . . . 5
  - 2.2.3 Spend-circuit divergence . . . 6

3 Data Flow: Mint **6**
- 3.1 Single-note mint . . . 6
- 3.2 Serialized-note mint (N sub-notes in one leaf) . . . 6
- 3.3 Mint-phase field origin + privacy (side-by-side) . . . 7

4 Data Flow: Spend **7**
- 4.1 Single-note spend . . . 7
- 4.2 Serialized-note spend . . . 7
- 4.3 Spend-phase field origin + privacy (side-by-side) . . . 7
- 4.4 Calldata payload comparison . . . 8

5 Storage Layout **9**
- 5.1 Single-note nullifier set . . . 9
- 5.2 Serialized-note per-parent bitmap . . . 9
- 5.3 Storage cost per note . . . 9

6 Per-field Security Analysis (consolidated) **10**

7 Attack Analysis **10**
- 7.1 Forging a fresh sub-note from a victim's payload . . . 10
- 7.2 Family linkage disclosure . . . 10
- 7.3 Double-spend . . . 11
- 7.4 Bit-flip "nullifier squat" . . . 11
- 7.5 Partial-batch recipient abandonment . . . 11
- 7.6 Brute-force over the payload space . . . 11

8 Cost Comparison **12**
- 8.1 Per-bearer-note mint cost . . . 12
- 8.2 Per-bearer-note spend cost . . . 12
- 8.3 Verifier bytecode . . . 12
- 8.4 Storage cost (amortized) . . . 12

9 Privacy Model **13**
- 9.1 What an on-chain observer sees . . . 13
- 9.2 What a recipient learns about siblings . . . 13
- 9.3 Why A1/A2 can't use serialization . . . 13

10 Migration Path 14
10.1 Phase A: build the spend-serialized circuit . . . 14
10.2 Phase B: contract support . . . 14
10.3 Phase C: wallet support . . . 14
10.4 Phase D: measure + promote . . . 14

11 Open Questions 14

12 Cross-References 15

1 Scope and Summary

1.1 Applicability

Note flavor	Single-note	Serialized	Reason
A1 (public identified)	yes	NO	family linkage at spend exposes issuer->payee graph
A2 (private identified)	yes	NO	same as A1; same-M account detection becomes trivial
B1 (bearer)	yes	YES	bearer transfer is already public; linkage is free

Serialization is strictly opt-in on a per-mint basis: the same issuer may mint some families as serialized B1s and others as single B1s or A1/A2s within the same batch (the batch-mint circuit is unchanged – see *Mint-Circuit Reuse*).

1.2 Headline differences

Axis	Single Note	Serialized B1 (N sub-notes)
Leaves per recipient	1	1 (shared with N-1 siblings)
Marginal mint gas / bearer	~31K (asymptotic)	~31K/N
Spend gas	~360K	~400-450K (+sub-tree Merkle walk)
Nullifier storage	20K SSTORE per spend	amortized ~80 gas / bit in bitmap
On-chain family linkage	none (cm hidden at spend)	`cm_parent` public on each spend
Brute-force margin	2^128 (birthday, Poseidon)	2^128 (same, plus sub-tree binding)
Mint circuit changes	(baseline)	NONE (see *Mint-Circuit Reuse*)
Spend circuit changes	(baseline)	+log_2(N) Merkle-walk levels

The rest of this document justifies each row.

2 Construction

2.1 Single Note (status quo)

Each commitment is an independent Poseidon-5 opening of five fields:

```
cm = Poseidon-5(flavor, v, rho, idHash, predicate)
```

At mint, `cm` is inserted at a fresh leaf of `noteRoot` (via the mint-batch SNARK, which may insert up to N such `cm` in one tx). At spend, the holder reveals:

```
nf = Poseidon(rho, idHash, 4242)
```

as a public input; the contract records `nullifiers[nf] = true` and transfers `v` BUCK to the recipient. `rho` is a ~254-bit issuer-chosen random; `nf` is therefore a uniform element of the scalar field conditional on `rho` being fresh, which makes it both double-spend-unique and unlinkable across an observer's view of two unrelated notes.

2.2 Serialized B1 sub-notes

For a family of N sub-notes (N = 2^K, K small – typical K in {4, 8, 10, 16} for N in {16, 256, 1024, 65536}), the issuer:

1. Draws a fresh per-family secret seed `K_fam`.
2. Generates `s_i = PRF(K_fam, i)` for i in 0..N-1. Each `s_i` is a ~254-bit field element, pseudorandom. `PRF` is domain-separated Poseidon or equivalent; `K_fam` is destroyed after step 4 and does not appear again anywhere on-chain or in any recipient payload.
3. Computes `subRoot` = depth-K Poseidon-2 Merkle root of `[s_0, ..., s_{N-1}]`.
4. Computes `predicate_fam = Poseidon-2(predicate_base, subRoot)`, where `predicate_base` is whatever policy field a single-note B1 would use.

   The parent commitment is then the *ordinary* Poseidon-5:

   ```
   cm_parent = Poseidon-5(flavor, v, rho, idHash, predicate_fam)
   ```

   – and the mint-batch circuit hashes `cm_parent` identically to any other B1 leaf. See *Mint-Circuit Reuse* below.
5. For each recipient of sub-note i, the issuer delivers (off-chain, via any bearer-note-appropriate channel):

   ```
   (flavor, v, rho, idHash, predicate_base, subRoot,
    i, s_i, path_i_in_subRoot, path_cm_parent_in_noteRoot)
   ```

   Note that `K_fam` is NOT in the payload; it has served its purpose and is destroyed. `predicate_base` and `subRoot` together let the recipient reconstruct `predicate_fam`, which in turn lets them reconstruct `cm_parent` and prove its membership in `noteRoot`.

2.2.1 How PRF-of-index becomes witness-and-verify

The spend circuit does *not* invert `s_i -> i`. That would require breaking the PRF, and would also let anyone else do it – which defeats the scheme (an attacker with one recipient's payload could enumerate every sibling's serial).

Instead, the recipient at spend time witnesses `(i, s_i, path_i_in_subRoot)` privately, and the circuit verifies the binding

```
MerkleOpen(subRoot, position = i, leaf = s_i, path = path_i_in_subRoot) = true.
```

`i` is emitted as a public signal; the contract uses `(cm_parent, i)` to flip the corresponding bit in `bitmap[cm_parent]`. The attacker cannot fabricate a fresh `(i', s_{i'}, path_{i'})` because `subRoot` is committed via `cm_parent` in `noteRoot`: producing an alternate triple requires either breaking Poseidon preimage resistance (2^128 work, birthday) or inverting the on-chain `noteRoot` binding (same).

2.2.2 Mint-circuit reuse

Because `cm_parent` is an ordinary Poseidon-5 output, the mint-batch circuit (`circuits/mint_batch.circom`) treats serialized and single B1 leaves identically. Only the *wallet's* pre-mint preparation differs: a single B1 computes `predicate = predicate_base`, while a serialized family computes `predicate = Poseidon-2(predicate_base, subRoot)` after building the sub-tree. No new pinned-N variants, no new trusted setup, no new verifier bytecode.

The cost of serialization to the mint path is therefore zero on-chain and O(N) hashes off-chain – a single-threaded laptop builds `subRoot` for N=1024 in <100ms.

2.2.3 Spend-circuit divergence

The spend circuit *does* change: it must

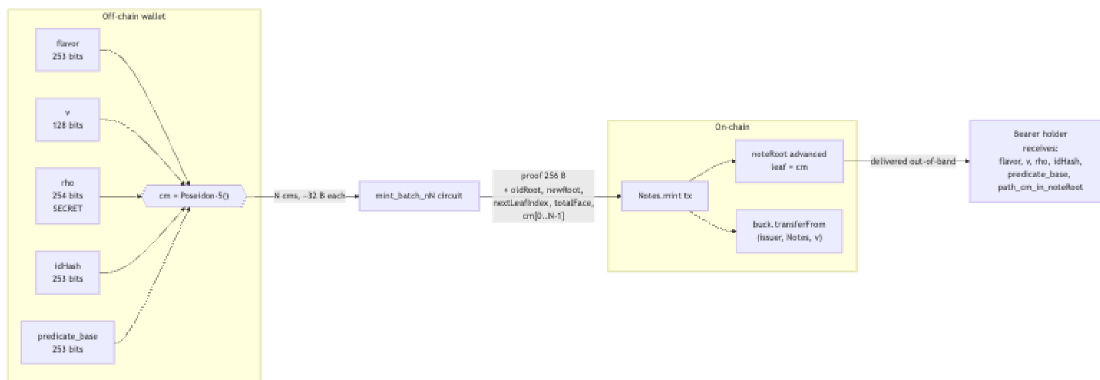
1. Open `cm_parent` from `(flavor, v, rho, idHash, predicate_base, subRoot)`.
2. Open `predicate_fam = Poseidon-2(predicate_base, subRoot)`.
3. Prove `cm_parent` in `noteRoot` (unchanged vs. single-note).
4. Prove `s_i` in `subRoot` at position `i` (NEW: depth-K Merkle walk).
5. Emit `(cm_parent, i)` as public signals instead of `nf`.

This is ~1.5-2x the constraint count of single-note spend at N=1024 (where K=10; the sub-tree walk adds 10 Poseidon-2 levels). We deploy it as a separate per-flavor verifier; the spend adapter dispatches on a flag or on the calldata shape.

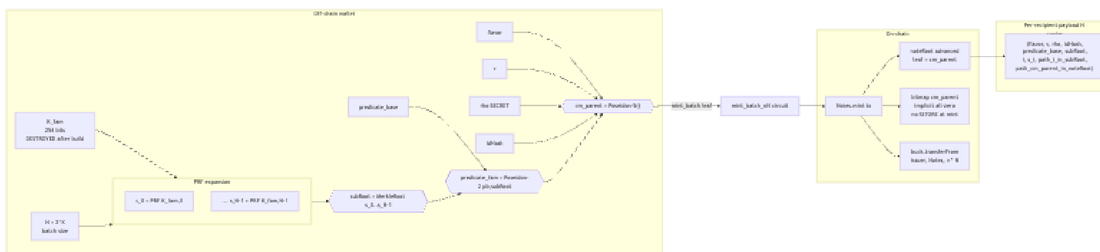
3 Data Flow: Mint

The mint path is nearly identical between the two variants; the only difference is in how the wallet computes `predicate` before handing `cm` to the mint-batch prover.

3.1 Single-note mint



3.2 Serialized-note mint (N sub-notes in one leaf)



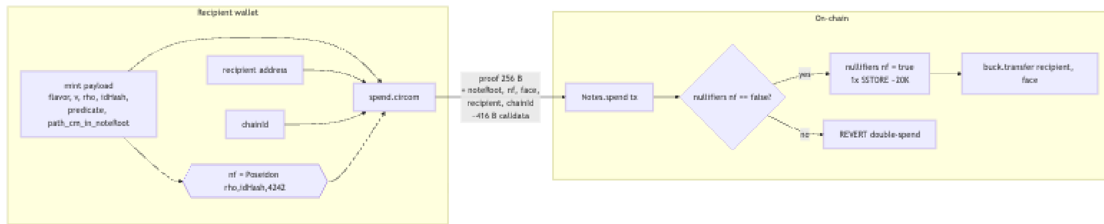
3.3 Mint-phase field origin + privacy (side-by-side)

Field	Single	Serialized	Source	Visibility at mint
`flavor`	present	present	issuer	in-circuit only
`v`	present	present	issuer	`totalFace` sum
`rho`	present	present	issuer random	in-circuit only
`idHash`	present	present	issuer	in-circuit only
`predicate_base`	`predicate`	present	issuer	in-circuit only
`K_fam`	absent	present	issuer random	NEVER exposed
`s_i`	absent	present	`PRF(K_fam, i)`	in-circuit only
`subRoot`	absent	present	Merkle(`s_0..s_{N-1}`)	in-circuit only (via `predicate_fam`)
`predicate_fam`	absent	present	`Poseidon-2(pb, subRoot)`	in-circuit only
`cm` / `cm_parent`	present	present	Poseidon-5 of above	public leaf in `noteRoot`

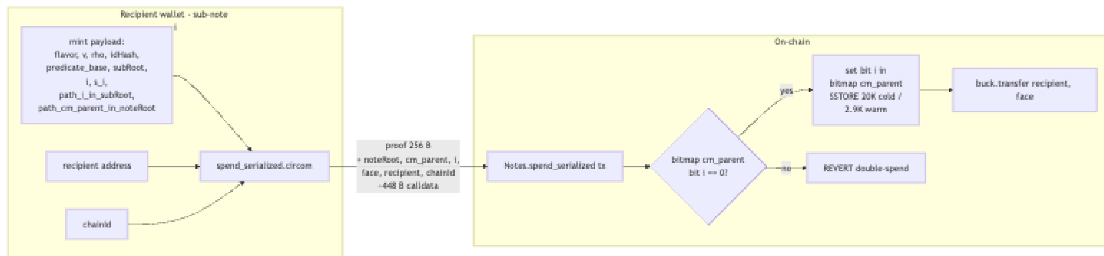
4 Data Flow: Spend

Here the two encodings diverge visibly.

4.1 Single-note spend



4.2 Serialized-note spend



4.3 Spend-phase field origin + privacy (side-by-side)

Field	Single	Serialized	Origin	Who sees on-chain	Attack if forged
`noteRoot`	public	public	contract state	everyone	stale-state rever
`nf`	public	absent	in-circuit from `rho,idHash`	everyone	double-spend (r
`cm_parent`	absent	public	recipient witness	everyone	SNARK rejects
`i`	absent	public	recipient witness	everyone	SNARK rejects
`face`	public	public	recipient witness	everyone	SNARK rejects
`recipient`	public	public	recipient witness	everyone	front-run (SNA
`chainId`	public	public	EIP-155	everyone	cross-chain repl
`rho`	private	private	issuer	in-circuit only	– (secret)
`idHash`	private	private	issuer	in-circuit only	– (secret)
`s_i`	absent	private	`PRF(K_fam, i)`	in-circuit only	SNARK rejects
`subRoot`	absent	private	mint-time	in-circuit only	– (bound via `cm`
`path_cm_in_noteRoot`	private	private	wallet	in-circuit only	– (bound via M
`path_i_in_subRoot`	absent	private	mint payload	in-circuit only	– (bound via `su`

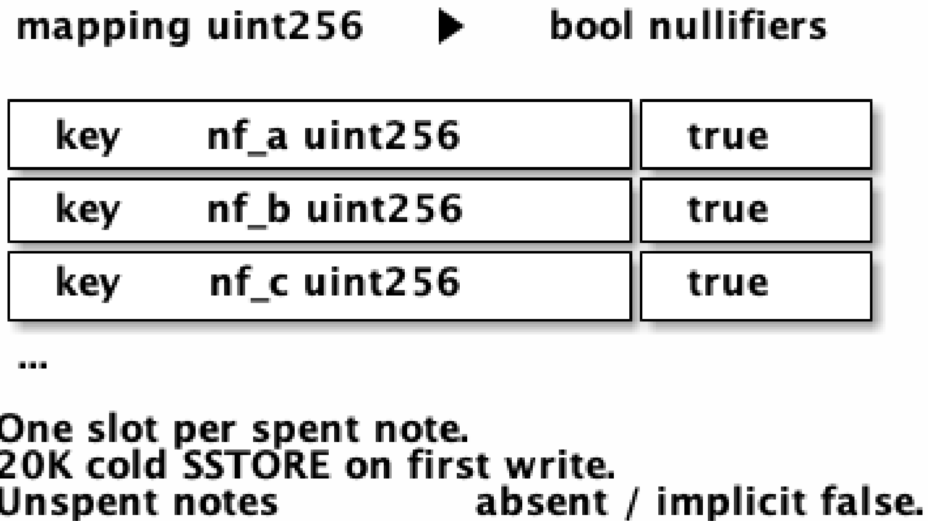
4.4 Calldata payload comparison

Item	Single	Serialized
Groth16 proof	256 B	256 B
`noteRoot`	32 B	32 B
`nf`	32 B	–
`cm_parent`	–	32 B
`i`	–	32 B
`face`	32 B	32 B
`recipient`	32 B	32 B
`chainId`	32 B	32 B
Total spend calldata	~416 B	~448 B
+ calldata-IC (if hash-pinned)	+~8 KB	+~10 KB

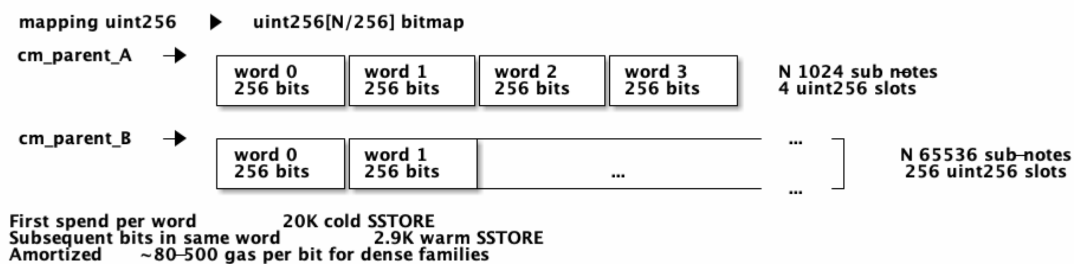
The 32 B delta (`i` as an extra public input) costs 512 gas at 16 gas/byte; trivial.

5 Storage Layout

5.1 Single-note nullifier set



5.2 Serialized-note per-parent bitmap



5.3 Storage cost per note

Family size	Words	Worst-case per-note storage	Amortized per-note storage (all spent)
Single (N=1)	1	20K (cold)	20K
Serialized N=16	1	20K first + 14*2.9K + final	~3.8K
Serialized N=256	1	20K + 255*2.9K	~2.98K
Serialized N=1024	4	4*20K + 1020*2.9K	~2.96K
Serialized N=65536	256	256*20K + 65280*2.9K	~2.96K

Serialized storage is asymptotically ~2.96K per sub-note (SLOAD + SSTORE warm), vs 20K for single – a ~7x reduction for fully-spent families. Partially-spent families (common case) save somewhat less, since each newly-touched word pays the cold penalty.

6 Per-field Security Analysis (consolidated)

Reading the tables together so the whole attack surface is on one screen:

Field	Phase	Role	Forgery defense
flavor, v, rho, idHash, predicate	mint	commitment opening	Poseidon preimage (2^12
K_fam	mint only	PRF seed for s_i	n/a (destroyed post-buil
s_i	mint+spend	sub-note serial	PRF + subRoot binding
subRoot	mint	commits s_0..s_{N-1}	Poseidon-5 cm binding
cm / cm_parent	mint+spend	noteRoot leaf	noteRoot Merkle bind
i	spend	bit index in family	subRoot Merkle bind
nf	spend	single-note double-spend key	Poseidon of rho,idHash
bitmap[cm_parent]	contract	serialized double-spend set	SSTORE guarded by bit-
path_cm_in_noteRoot	spend	anchor to on-chain state	noteRoot bind
path_i_in_subRoot	spend	anchor to family	subRoot bind
face / recipient / chainId	spend	binding to tx	SNARK public inputs

7 Attack Analysis

7.1 Forging a fresh sub-note from a victim's payload

Attacker holds recipient i's full payload and wants to produce a valid spend for j ! i= without ever having received s_j. They would need either:

1. A second preimage s_j' ! s_j= of subRoot at position j that the circuit accepts. This is a Poseidon preimage attack against a committed tree: 2^128 work by birthday (Poseidon-2 is 254-bit output, standard security bound).

2. Inverting s_j = PRF(K_fam, j) without K_fam – infeasible for any PRF worth the name. Even if PRF were keyless, the attacker would still need to open subRoot at position j, which they don't control.

The attack cost is therefore bounded below by the Poseidon preimage bound (2^128), identical to what protects a single-note cm from forgery. Serialization adds zero attack surface at the "forge a note I don't own" level.

7.2 Family linkage disclosure

Two serialized spends sharing the same cm_parent are observably related: an on-chain observer sees cm_parent as a public input to both spend txs and can bin-sort spends by family. Consequences:

- For B1 bearer notes: neutral. Bearer transfer doesn't carry identity; observing that two bearers happen to hold siblings of the same mint leaf discloses nothing beyond "both obtained notes of the same denom in the same family". The issuer is already publicly identified at mint (transferFrom has msg.sender), and the denom face is a public input anyway. The family boundary is not a new leak.

- For A1/A2 notes: unacceptable. Linking spends under cm_parent binds unrelated payees to one mint, defeating the A2 privacy property.

This is exactly the scope boundary in *Applicability*: serialized encoding is B1-only because B1 is the only flavor that doesn't care about family linkage.

7.3 Double-spend

Single-note: `nullifiers[nf]` is an SSTORE-set gate; the second spend of a given note finds the slot already true and reverts. Pre-image freshness of `nf` under `rho,idHash` is the cryptographic guard.

Serialized: `bitmap[cm_parent][i/256] bit (i % 256)` is the SSTORE-set gate; the second spend with the same `(cm_parent, i)` finds the bit already set and reverts. The `(cm_parent, i)` tuple uniquely identifies the sub- note; forging a different `(cm_parent, i')` that maps to the same bit requires breaking `subRoot` binding (see *Forging* above).

Both models close under double-spend by the same mechanism (on-chain set-once guard on a SNARK-validated coordinate); the single-note model uses a 256-bit coordinate, the serialized model a 256+log2(N)-bit coordinate.

7.4 Bit-flip "nullifier squat"

Could Mallory flip a bit in `bitmap[cm_parent]` for a sub-note she does NOT own, pre-emptively blocking the real recipient's spend? No: the only code path that writes the bitmap is `Notes.spend_serialized`, which itself requires a Groth16 proof binding `(cm_parent, i)` to a valid `s_i` and recipient. Mallory cannot produce such a proof for a sub-note she doesn't hold (see *Forging*), and spending the sub-note she holds transfers the funds to *her* recipient address – which is equivalent to legitimately receiving a B1 and spending it, not an attack.

The only residual "attack" is an out-of-band: Mallory may refuse to forward (or delay) a payload to its intended recipient. That's a trust issue between issuer and payee, unchanged from any single-note B1 courier model.

7.5 Partial-batch recipient abandonment

If the issuer mints N=1024 sub-notes intending to distribute all of them, but only distributes 800, the remaining 224 sit unspendable in the bitmap forever (their `s_i` are known only to the issuer, who presumably destroyed `K_fam`). On-chain footprint: zero (unspent bits cost nothing). Issuer impact: paid `v * N` BUCK at mint but only delivered to 800 recipients; the 224 undelivered "slots" represent a permanent burn of `v * 224` BUCK.

This is the same economic self-punishment as minting a single note one never intends to spend – no new failure mode.

7.6 Brute-force over the payload space

Attacker has *no* valid recipient payload, but wants to guess one. Two paths:

1. Guess `(cm_parent, i, s_i, paths)` such that the SNARK accepts and the bit is unset. The SNARK rejects everything except valid openings; to produce a valid opening attacker must find a Poseidon preimage of some `cm_parent` already in `noteRoot` – i.e., break the commitment scheme (2^128 birthday).

2. Guess `(i, s_i, paths)` compatible with a specific `cm_parent` they observed from a prior spend. The `subRoot` inside `cm_parent` pins exactly N valid `(i, s_i)` pairs; attacker must find a second (i, s') that opens at the same position – Poseidon-2 second preimage, 2^128.

Both paths land at the same security level the single-note scheme enjoys (Poseidon 128-bit security). Serialization does not weaken brute-force resistance.

8 Cost Comparison

8.1 Per-bearer-note mint cost

Numbers from `test/MintVerifier.t.sol`, affine model `mint_tx_gas(N_batch) = 125K + 31K * N_batch` from `alberta-buck-notes-rollup-mint.org`. "N_fam" below is the serialization depth within one family (parent); "N_batch" is the number of parents per mint tx. Taking N_batch=16 as the representative batch:

Scheme	Notes delivered per tx	Gas per tx	Per-note gas
Shipped per-leaf insertion	2	~1.9M	~950K
Single-note (pivot, N_batch=16)	16	624K	39K
Serialized N_fam=16, N_batch=16	256	624K	2.4K
Serialized N_fam=256, N_batch=16	4096	624K	152
Serialized N_fam=1024, N_batch=16	16384	624K	38
Serialized N_fam=65536, N_batch=16	1048576	624K	0.6

Serialization is a pure off-chain prover cost (N_fam Poseidon-2 hashes to build `subRoot`); on-chain cost is unchanged from single-note mint at the same `N_batch`. Per-note mint cost drops with the batch-mint pivot by ~25x, and then with serialization by another =N_fam=x.

8.2 Per-bearer-note spend cost

Scheme	Proof verify	Merkle walks in-circuit	Storage write	Est. tx gas
Single-note	~200K	1 x TREE_DEPTH=20	1 SSTORE (20K cold)	~360K
Serialized N_fam=16	~220K	1 x 20 + 1 x 4	1 SSTORE (20K cold)	~390K
Serialized N_fam=256	~225K	1 x 20 + 1 x 8	1 SSTORE (20K/2.9K)	~400K
Serialized N_fam=1024	~230K	1 x 20 + 1 x 10	1 SSTORE (20K/2.9K)	~410K
Serialized N_fam=65536	~250K	1 x 20 + 1 x 16	1 SSTORE (20K/2.9K)	~440K

Spend cost grows logarithmically in `N_fam` (one extra Merkle level = ~4K gas in Groth16 verify overhead + ~600 gas in proof-gen scaling). Even for N_fam=65536, spend tx is ~440K, a ~20% increase over single-note.

8.3 Verifier bytecode

The spend-serialized verifier is a distinct circuit from single-note spend. Measured = TODO – the circuit is unimplemented as of 2026-04-23; current work is in `scripts/snark/serialized_notes_poc.py` (Python driver for the construction, N up to 4096 validated). Projected constraint count: ~25K for N_fam=1024 (single-note spend baseline + 10-level Poseidon-2 sub-tree walk); the resulting Solidity verifier should fit well under the 24 KB EIP-170 ceiling.

8.4 Storage cost (amortized)

Per the *Storage Layout* section: serialized amortizes to ~2.96K per sub- note (one SLOAD + one warm SSTORE), vs 20K for single. The gain compounds for issuers minting many bearer families –

a 1024-note B1 family costs ~3M gas to fully spend vs ~20M for 1024 single-note B1s, even before factoring the per-note mint savings.

9 Privacy Model

9.1 What an on-chain observer sees

Event	Single-note spend	Serialized spend
Calldata `noteRoot`	public (pins to recent block)	same
Calldata `nf`	pseudorandom, unlinkable	ABSENT
Calldata `cm_parent`	ABSENT	public – family boundary revealed
Calldata `i`	ABSENT	public – position-in-family revealed
Calldata `face`	public	same (denom disclosed in both)
Calldata `recipient`	public	same (payee disclosed in both)
Event `Spent`	face, recipient	face, recipient, cm_parent, i
Storage write	one new `nullifiers[nf]` slot	one bit in `bitmap[cm_parent]`

Serialized spends link one additional way: any two spends sharing `cm_parent` came from the same mint. Unique to that encoding; the main reason A-flavored notes must not use it.

9.2 What a recipient learns about siblings

A serialized-note recipient knows:

- They hold position `i` in a family of size `N`.
- `N` is plaintext in the payload (it's log2 of the `path_i_in_subRoot` length).
- `s_i` is theirs; they know *none* of the other `s_j` (PRF-protected).
- Once any sibling spends, `cm_parent` becomes publicly linked to some spent bit; by observing other spends under the same `cm_parent` the recipient can count which positions have gone through, but cannot tell *who* held them (unless `recipient` address correlations leak externally).

A single-note recipient learns nothing about any other note in the same mint beyond what `face` and `noteRoot` already reveal publicly.

9.3 Why A1/A2 can't use serialization

A1 (public identified): recipient at spend binds `idHash` to a registered identity. If two A1 sub-notes of the same family are spent by different payees, the linkage between their identities is on-chain. This leaks the issuer's entire "payees of this batch" graph and defeats the partial- fungibility property A1 is meant to provide.

A2 (private identified): same leak, one layer removed via ElGamal. If same-M detection is performed across siblings, the encrypted identifiers cluster under the shared family, which an adversary can use to correlate.

B1 (bearer): no identity attached at all. The only disclosure is denom and family membership, neither of which is a secret in a bearer model.

10 Migration Path

10.1 Phase A: build the spend-serialized circuit

1. `circuits/spend_serialized.circom` – extend `circuits/spend.circom` with the `(subRoot, i, s_i, path_i_in_subRoot)` sub-block.
2. Trusted setup under `scripts/snark/setup.sh` (fits under pot17-pot19 per constraint count, re-using existing ptau).
3. `src/SpendSerializedGroth16Verifier.sol` + per-flavor adapter (`src/SpendVerifierAdapter.sol` already supports multi-verifier routing; just add a "serialized" registration).

10.2 Phase B: contract support

1. `Notes.sol` adds `spendSerialized(proof, oldRoot, cm_parent, i, face, recipient)` alongside existing `spend(proof, ..., nf, ...)`.
2. New storage `mapping(uint256 => mapping(uint256 => uint256)) bitmap` (parent => word => 256 bits).
3. Keep `nullifiers` mapping unchanged for single-note and A1/A2 flows.

10.3 Phase C: wallet support

1. Mint path: add "serialize with N" option that builds `K_fam`, `subRoot`, `predicate_fam`.
2. Payload schema v2: extend the bearer-note blob format to include `(i, s_i, path_i_in_subRoot, subRoot)` when the note is serialized.
3. Spend path: detect payload variant from schema version, route to the appropriate verifier.

10.4 Phase D: measure + promote

1. Measure spend-serialized gas at N_fam in {16, 256, 1024, 65536}.
2. Confirm verifier bytecode size; confirm no EIP-170 breach.
3. Document family-linkage disclosure prominently in wallet UX.
4. Consider whether the issuer-side cost of destroying `K_fam` securely (HSM attestation?) warrants a protocol-level requirement.

11 Open Questions

1. ***PRF choice for*** `s_i`. Plain domain-separated Poseidon `PRF(K_fam, i) = Poseidon-2(K_fam, i)` is the obvious choice. Poseidon keyed-mode security is well-studied for 128-bit output; do we want a second hash (e.g., BLAKE3) as independent fallback in the wallet, or accept the Poseidon-only monoculture?
2. ***Destroy-`K_fam`- discipline***. The construction depends on the issuer not retaining `K_fam` past mint-time. If the issuer keeps `K_fam` they can impersonate any recipient (produce `(i, s_i, path)` on demand). Enforceable only by wallet UX / HSM attestation.

3. */Per-family vs. global bitmap/. We chose per-parent `mapping(cm => uint256[])` for locality; an alternative is a single global sparse bitmap keyed by `H(cm_parent, i)`. Global bitmap drops the `cm_parent` indirection (1 SLOAD instead of 2) at the cost of spreading writes across the full storage space (all SSTOREs cold). Break-even at ~8 sub-notes per family – not attractive for small N_fam.

4. ***Batched-spend per family***. Can a single recipient spend multiple sub-notes of one family in one tx? Useful for *aggregation* by a pooling payee. Straightforward circuit extension: M parallel `(i_j, s_{i_j}, path_{i_j})` openings under one `cm_parent` binding, emitting M bits to flip. Storage: M warm SSTOREs into the same `bitmap[cm_parent]` words – very cheap.

5. */Hiding/ `N_fam`. Currently `N_fam` is recoverable from the `path_i_in_subRoot` length at spend time (10 levels -> N=1024). Can we pad all serialized spends to a fixed max depth (say K=16 -> N_max=

   (a) and always emit 16-level paths? Prover cost +6 hashes per

   spend; anonymity gain is that an observer cannot distinguish N=16 families from N=65536 families. Probably worth it if serialization sees significant deployment.

6. ***POC-to-production port***. `scripts/snark/serialized_notes_poc.py` implements the full construction in ~250 lines of Python including a forgery-attempt test driver. Porting to `circuits/spend_serialized.circom` is the remaining implementation work.

12 Cross-References

- *Phase 7-bis batch-mint design*: `alberta-buck-notes-rollup-mint.org` (cost model, pinned-N policy, EIP-170 wall, calldata-IC). Serialization reuses the mint circuit from that document unchanged.

- *Formal proofs*: `alberta-buck-proofs.org`, Theorem 7 (mint soundness), Theorem 8 (spend soundness), Theorem 9 (double-spend resistance). A Theorem 12 for serialized spend is queued.

- *POC driver*: `scripts/snark/serialized_notes_poc.py` implements issuer-mint + recipient-verify + spend-predicate + forgery tests at N_fam in {256, 1024, 4096}. All tests pass; brute-force margin ~2^128.